Graph cuts based active contour model with selective local or global segmentation

Q. Zheng, E.Q. Dong and Z.L. Cao

Graph cuts (GC) and the active contour model (ACM) have become two of the most important schemes in image segmentation. Recently, many researches tend to unify the two schemes to obtain new models for efficient calculation and global minimisation. However, the existing GC based ACMs not only cannot achieve local segmentation, but also suffer from the determination of the regularising parameter, which is used to balance the edge and region terms in the existing GC based ACMs. Proposed is a new GC based ACM (NGC-ACM) to solve the two problems above. First, a new energy function without the regularising parameter is proposed for segmentation, which avoids the edge and region balance problem. Secondly, through constructing a specified graph, the proposed model can achieve selective local or global segmentation, which not only can extract all the objects globally, but also can extract the desired object locally. Experiments on synthetic and real images demonstrate the advantages of the proposed NGC-ACM over the existing GC based ACMs like GC based geodesic active contours with region forces (GC-GACWRF) in solving the two problems above.

Introduction: The active contour model (ACM) is one of the most successful variational models in image segmentation. Until now, there have been many studies on ACMs because they can easily combine different information such as boundary, region and shape prior. In this Letter, we consider the geodesic active contours with region forces in [1]:

$$E(C) = \mu \times \int_C g_b(|\nabla I(C(s))|)ds + \left(\int_{C_{\text{in}}} (I(x) - c_{\text{in}})^2 dx + \int_{C_{\text{out}}} (I(x) - c_{\text{out}})^2 dx\right) \tag{1}$$

where C is the evolving curve, μ is the regularising parameter used to balance the region and edge terms. c_{in} and c_{out} are mean intensities inside and outside C respectively.

Graph cuts (GC) [2, 3] is a powerful combinatorial optimisation method for solving discrete labelling problems by the minimising energy function in the following standard form [2]:

$$E(f) = \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N} V_{p,q}(f_q, f_q) \tag{2}$$

In (2), $f: P \to L$ is a labelling function, where P and L are sets of pixels and labels, respectively. N is a neighbourhood system. The terms $D_p(f_p)$ and $V_{pq}(f_p, f_q)$ measure the penalty of assigning label f_p to pixel p and the penalty of assigning labels f_p, f_q to the pixels p, q, respectively.

Recently, many researches tend to use GC to optimise ACMs [4] because of more efficient calculation and global minimisation than the level set method, and the goal is to find a labelling which can minimise the discrete energy function like (2). In [1], the discrete formulation of energy (1) has been given as follows:

$$\begin{aligned} E &= \mu \times E_b(p,q) + E_r(p) \\ &= \mu \times \sum_{p,q \in \Omega} \sum_{p \in N(q)} \frac{\omega_{pq} \times ((1 - x_p)x_q + x_p(1 - x_q))}{1 + \beta|I(p) - I(q)|} \\ &\quad + \left(\sum_p (I(p) - c_s)^2 (1 - x_p) + \sum_p (I(p) - c_t)^2 x_p\right) \end{aligned} \tag{3}$$

where μ is the regularising parameter as in energy (1), $\beta > 0$, pixel p is in the vicinity of pixel q, i.e. $N(q)$, and $\omega_{pq} = \delta^2 \Delta\theta_{pq}/|e_{pq}|$ with the angular differences $\Delta\theta_{pq}$ between the nearest edge lines and the length $|e_{pq}|$ of edge e_{pq}. c_s and c_t can be calculated by (4) based on the corresponding relations in Table 1:

$$\begin{aligned} c_s &= \left(\sum\nolimits_p I(p)(1 - x_p)\right) \Big/ \left(\sum\nolimits_p (1 - x_p)\right) \\ c_t &= \left(\sum\nolimits_p I(p)x_p\right) \Big/ \left(\sum\nolimits_p x_p\right) \end{aligned} \tag{4}$$

However, there are another two problems in GC based ACMs with the form of energy (3). 1. GC based ACMs are a global segmentation model, but sometimes local segmentation of the desired object from the background is necessary in practical applications. 2. The regularising parameter μ in energy (3), which is used to balance the edge and region terms, often influences the segmentation results greatly, and the determination of the regularising parameter μ is difficult. In this Letter, we propose a new GC based ACM to solve the two problems above.

Table 1: Corresponding relations of source S / sink T, x_p (1/0) and background ('B') / object ('O')

S / T	x_p (1/0)	B / O
S	0	B
T	1	O

Proposed GC based ACM: In this Letter, in order to avoid the region and edge balance problem and obtain the property of selective local or global segmentation, we propose a new GC based ACM without the regularising parameter μ in (5) and construct a specified graph for selective local or global segmentation:

$$E = E_b(p,q) \times E'_r(p,q) \tag{5}$$

In (5), $E_b(p,q)$ has the same representation as in energy (3), and $E'_r(p,q)$ indicates the penalty for assigning pixels p and q different labels as (6):

$$\begin{aligned} E'_r = \sum_p \min\,&(((I(p) - c_s)^2 + (I(q) - c_t)^2), \\ &((I(p) - c_t)^2 + (I(q) - c_s)^2)) \end{aligned} \tag{6}$$

where c_s and c_t can be calculated by (4) based on Table 1.

The selective local or global segmentation property can be obtained by constructing a specified graph based on Table 1 as follows:

- n-links (neighbourhood links): The weights of edges connecting neighbouring pixel nodes are $E_b(p,q) \times E'_r(p,q)$.
- t-links (terminals S/T links): The property of selective local or global segmentation is determined by the selection of the seed pixel nodes. Local segmentation can be obtained when the T-connected seed pixel nodes only locate in the desired object, and global segmentation can be obtained when the T-connected seed pixel nodes locate in all of the objects. For convenience, we give the following definition for each pixel p.

$$\begin{aligned} &\Phi_s(p) = (I(p) - c_s)^2, \quad \Phi_t(p) = (I(p) - c_t)^2, \\ &\Delta\Phi(p) = \frac{|\Phi_s(p) - \Phi_t(p)|}{2} \end{aligned} \tag{7}$$

For global segmentation, if $\Phi_s(p) > \Phi_t(p) + \Delta\Phi(p)$, then add link pT with weight $9e + 9$, and if $\Phi_t(p) > \Phi_s(p) + \Delta\Phi(p)$, then add link Sp with weight $9e + 9$.

For local segmentation of non-annular objects, if $\Phi_s(p) > \Phi_t(p) + \Delta\Phi(p)$ and $x_p = 1$, then add link pT with weight $9e + 9$, and if $\Phi_t(p) > \Phi_s(p) + \Delta\Phi(p)$ and $x_p = 0$, then add link Sp with weight $9e + 9$.

For local segmentation of annular objects, if $\Phi_s(p) > \Phi_t(p) + \Delta\Phi(p)$ and $x_p = 1$, then add link pT with weight $9e + 9$, and if $\Phi_t(p) > \Phi_s(p) + \Delta\Phi(p)$, then add link Sp with weight $9e + 9$.

Therefore, the algorithm of the proposed model can be summarised as follows:

1. Initialise curve C anywhere in the image for global segmentation, or close to the desired object for local segmentation. Initialise the binary variable $x_p = 1$ if p is inside C, and $x_p = 0$ if p is outside C.
2. Calculate c_s and c_t using (4).
3. Construct the graph for the proposed energy (5) as described above.
4. Calculate the minimum cut on the constructed graph by the max-flow/min-cut algorithm in [5], and the graph G will be partitioned into partitions G_0 and G_1, where the source terminal $S \in G_0$ and the sink terminal $T \in G_1$.
5. Update the binary variable x_p as: $x_p = 0$ if $p \in G_0$, and $x_p = 1$ if $p \in G_1$.
6. Smooth the evolution curve with median filter.
7. Repeat steps 2–6 until convergence.

Experimental results: In GC based ACMs, the advantages of efficient calculation and global minimisation are obvious because of GC optimisation. Therefore, in this Section, we only take experiments to demonstrate the advantages of the proposed model in the aspects of the selective local or global segmentation property and avoiding the region and edge balance problem. For convenience, we name the proposed model NGC-ACM (new GC based ACM) in [1] GC- GACWRF (GC based geodesic active contours with region forces), the energy (3) which does not consider the narrow band.

Fig. 1 demonstrates the selective local or global segmentation property of the proposed NGC-ACM. Fig. 1*a* demonstrates the global segmentation property of the proposed NGC-ACM with arbitrary initialisation. Fig. 1*b* demonstrates the local segmentation property of the proposed NGC-ACM corresponding to annular and non-annular objects with initialised contours close to the desired object.

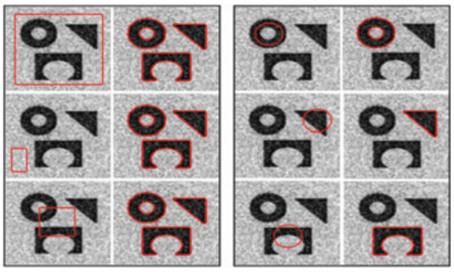

Fig. 1 *Selective local or global segmentation property of proposed NGC-ACM*

a Global segmentation by proposed NGC-ACM
b Local segmentation by proposed NGC-ACM

Fig. 2 shows the applications in medical images segmentation, and demonstrates influence of the regularising parameter μ to GC-GACWRF. The first row shows the ultrasound image of the left ventricle, the second row shows the magnetic resonance image of the left ventricle, and the third row shows the magnetic resonance image of the corpus callosum. Fig. 2*b* with $\mu = 0.1$ and Fig. 2*c* with $\mu = 0.5$ demonstrate that the regularising parameter μ has great influence on segmentation results in GC-GACWRF. Comparing Figs. 2*b* and *c* using GC-GACWRF, the proposed NGC-ACM in Fig. 3*d* can avoid the region and edge balance problem effectively, and the contours of the medical images can be accurately detected by the proposed NGC-ACM locally.

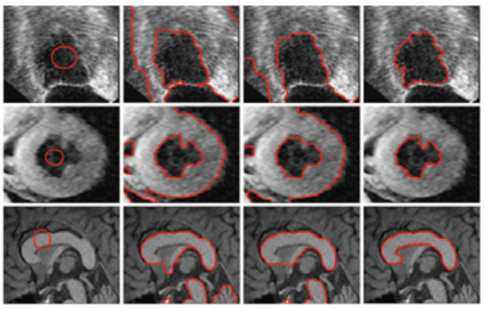

Fig. 2 *Applications in medical images segmentation*

a Initialised contours
b Segmentation by GC-GACWRF with $\mu = 0.1$
c Segmentation by GC-GACWRF with $\mu = 0.5$
d Segmentation by proposed NGC-ACM

Conclusion: In this Letter, we propose a new GC based ACM, which not only can avoid the region and edge balance problem, but also can achieve selective local and global segmentation. First, the new GC based ACM formulates a new discrete energy without the regularising parameter. Then a specified graph is constructed to guarantee the selective local or global segmentation. Finally, the minimum cut on the graph is computed by a max-flow/min-cut algorithm such that the minimisation of the proposed energy can also be obtained. The new GC based ACM is implemented iteratively, and experimental results demonstrate that the proposed model can achieve more flexible segmentation and be popularly used for image segmentation.

Acknowledgment: This work was supported by the Natural Science Foundation of Shandong Province (grant 2009ZRB01661).

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13 February 2012
doi: 10.1049/el.2012.0470
One or more of the Figures in this Letter are available in colour online.

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