Kullback-Leibler distance and graph cuts based active contour model for local segmentation

Wenyan Sun\textsuperscript{a,b}, Enqing Dong\textsuperscript{a,*}

\textsuperscript{a} School of Mechanical, Electrical and Information Engineering, Shandong University (Weihai), Weihai 264209, China
\textsuperscript{b} School of Information Engineering, Shandong University of Political Science, Jinan 250103, China

A R T I C L E   I N F O
Article history:
Received 19 June 2018
Received in revised form 20 January 2019
Accepted 6 April 2019

Keywords:
Active contour model
Graph cuts
Image segmentation
Kullback-Leibler distance

A B S T R A C T
Graph cuts based active contour model can obtain the global optimal solution and run faster. To achieve robust local segmentation of inhomogeneous images and improve operation efficiency, a modified Kullback-Leibler distance and graph cuts based active contour (KLD-GCBC) model in the iterative narrow band frame is proposed in this paper. The dynamic narrow band is constructed between the morphological erosion of evolving curve and the preset outer boundary. In the boundary term of discrete energy formulation, an adaptive coefficient is designed to adjust the weights of edges connecting adjacent pixels. And the Kullback-Leibler distance between the inner and outer local neighborhoods is introduced to the region term as the weight of n-links connecting pixels with terminal nodes. Experimental results on synthetic and medical images with intensity inhomogeneity show that the proposed model can make full use of local features of the image and achieve local segmentation more efficiently.

© 2019 Published by Elsevier Ltd.

1. Introduction
Active contour model (ACM) has been proved to be an effective image segmentation method and widely used, which can get closed and smooth boundary through the evolution of an initial contour using energy minimization theory. The desired segmentation result is determined by the optimal solution of minimizing the energy functional which is used to describe the contour and its evolution. The total energy functional is generally composed of internal and external energy, which can include the length, area or curvature of the evolving contour and the mean, variance or gradient of the image intensity. According to whether the energy functional contains free parameters, ACMs are divided into parametric models \cite{1,2} and geometric models \cite{3-6}. After the Snake model \cite{1} proposed in 1985, lots of classic and modified models \cite{7-9} have emerged, which has greatly promoted the development of ACM. By means of variation principle and gradient descent flow, the solution to the minimum of energy functional can be solved. But when the energy is not convex, the solution may not be global optimal, and thus cannot get correct segmentation result.

Graph cuts (GC) \cite{10} is a combinatorial optimization algorithm for solving the problem of energy minimization, which has been applied in image segmentation \cite{11}, image matting \cite{12}, stereo vision \cite{13} and so on. On the basis of graph theory, an image is mapped to a weighted graph and interactive operations of the user are needed to specify some seed nodes as source node s and sink node t corresponding to foreground or background respectively, which can be a single pixel or a collection of some pixels. Use $G=(V, E)$ to represent the undirected graph mapping to the image, $V = \{v\} \cup \{s, t\}$ to represent all nodes in G, and $E = \{e\} \cup \{(s, v) \cup (v, t)\}$ to denote all edges connecting two nodes in G, where $\{v\}$ is the collection of pixels to be classified, $\{e\}$ denotes all edges connecting adjacent nodes in the neighborhood system, called n-links, and $\{(s, v) \cup (v, t)\}$ represents the edges connecting any node v with the source node s and sink node t, called t-links. Each edge is assigned a non-negative weight.

Image segmentation can be seemed as a problem of binary labeling via GC, each node in G is set a label indicating whether it belongs to the foreground or background. Any cut of the graph can divide all pixels into two categories, and the sum of weights of all cut edges is called the cost of the cut. The cut with the minimum cost is the optimal segmentation result, which is also equivalent to the maximum flow. At present, the commonly used algorithms to achieve max-flow/min-cut mainly include Goldberg-Tarjan \cite{14} and Ford-Fulkerson \cite{15}. Then image segmentation is considered as minimizing the cost function. The prior knowledge, grayscale, and statistical information of the image can be used to define data term and smooth term of the energy, which correspond to regional constraint and boundary constraint, respectively.

ACM and GC are main methods of image segmentation and have achieved varying degrees of development. In recent years, GC is combined with ACM to overcome local minimum of traditional...
ACM, obtain global optimal solution of the energy function, and improve the operating efficiency, which has attracted extensive attention and become a research direction of image segmentation method. Several typical ACM models have been discretized and optimized globally using graph cuts. Boykov and Kolmogorov [16] used GC to find the global minimum of the energy function of geodesic active contour (GAC) model [3], which mainly uses the gradient of image and has the risk of falling into weak edges. Then some global or local region-based ACMs [18-20] via GC were proposed to improve the performance. Different from constructing global graph, Xu et al. [21] proposed a graph cuts based active contour (GCBC) model, and then the framework to find the global minimum within the contour neighborhood was introduced. And Tao [22] generalized the GCBC model and proposed three algorithms based on iterative narrowband and GC to optimize the GAC with region forces (GACRF) model for interactive object segmentation. In Ref. [23], global or local segmentation can be implemented by constructing different network graphs, but the proposed method is not applicable for inhomogeneous images for making use of global intensity information. To reduce the effect of the balance coefficient between edge term and data term in additive energy functional, some multiplicative GC based ACMs [24-26] were proposed to achieve local segmentation of inhomogeneous images, which can get better performance than additive models, but the stability and operational efficiency need to be improved.

Kullback-Leibler (KL) distance [27], also called relative entropy, is a metric to measure the difference between two probability distributions in the same event space. The closer two distributions are, the smaller the KL distance is. This measure plays an important role in information theory, and it has been proved that KL distance can be introduced to the energy term and used to object tracking [28,29], image segmentation [30-32] and pattern recognition [33]. In Ref. [30], Zheng et al. proposed an improved CV model using KL distances as the coefficients of internal and external energies, which can automatically adjust the parameters and get high robustness and efficiency. Cheng et al. [31] also proposed a region-based multi-phase level set method utilizing the between-cluster KL distance to formulate the energy function.

To achieve robust local segmentation of inhomogeneous images efficiently, a KL distance and graph cuts based active contour (KLD-GCBC) model is proposed in this paper, which constructs the network graph in an iterative narrowband framework, and designs the weights of n-links and t-links using local mean and statistical information.

The rest of this paper is organized as follows: Section 2 presents several classic active contour models based on GC. Section 3 presents the proposed KLD-GCBC model in detail. Section 4 takes experiments to validate the performance of KLD-GCBC model, and the paper is summarized in Section 5.

2. Related work

2.1. Graph cut based active contour without edges

In Ref. [19], a graph cut based Chan-Vese [4] model with relaxed homogeneity constraint was proposed, we name it as GCBCV, in which the discrete energy formulation of was presented as

\[ E = \mu \sum_{p,q \in E} \alpha_{pq} (x_p (1 - x_q) + x_q (1 - x_p)) + \sum_p |m(p) - m_0(p)|^2 x_p \]

\[ + \sum_p |im(p) - m_0(p)|^2 (1 - x_p) \]

where, \( im \) is the image, \( p \) denotes any pixel in the image with a binary variable \( x_p \), \( im(p) \) is the intensity of \( p \), \( m_i(p) \) and \( m_0(p) \) are the mean intensity in the neighborhood \( W \) of the pixel \( p \) inside and outside the contour, respectively, which are defined as

\[ m_i(p) = \frac{\sum_{p \in W} im(p) x_p}{\sum_p x_p} \]

\[ m_0(p) = \frac{\sum_{p \in W} im(p)(1 - x_p)}{\sum_p (1 - x_p)} \]

In Eq. (1), the first sum is the discrete representation of the length of the evolving contour, \( \mu \) is the balance coefficient of the edge item, \( \alpha_{pq} \) represents the edge connecting nodes \( v_p \) and \( v_q \) in the 8-neighborhood system, and \( \alpha_{pq} = \frac{\beta x_p}{\sqrt{\sum p x_p}} \) is the capacity of the edge \( e_{pq} \). In the construction of the graph, the weights of edges connecting each pixel \( p \) with terminal node \( S \) or \( T \) are assigned according to the difference of \( |im(p) - m(p)| \) and \( |im(p) - m_0(p)|^2 \).

2.2. GCBC

The GCBC [21] model regards the ideal segmentation curve as a global optimal solution of the width-known contour neighborhood (CN), which is derived from the morphological expansion of initial contour. The inner boundary of CN serves as the source point \( s \), and the outer boundary corresponds to the sink point \( t \). The pixels within the CN are mapped into a graph. The multi-source multi-sink problem is converted to single-source single-sink problem, and the graph is cut using the maximum flow/minimum cut algorithm. By continuously updating the CN and finding the global optimal solution within the CN until the algorithm converges, the final segmentation result is obtained.

GCBC is proved to converge after a finite number of iterations or oscillate back and forth between several cuts with same capacity. In the construction of network graph, GCBC requires high connectivity and edge weight settings for the nodes. The cost function of GCBC has not internal energy, and the weight of edge connecting adjacent nodes is assigned as follows:

\[ c(i,j) = (g(i,j) + g(j,i))^6 \]

where \( g(i,j) = exp(-grad_j|/max_k(grad_j|k)) \), and \( grad_j(k) \) is the intensity gradient at location \( k \) in the direction of \( i \rightarrow j \).

2.3. IINBBGC method

In Ref. [22], an improved iterative narrow band graph cuts (IINBBGC) method to solve the minimization of the GACRF model was proposed, which provides the general framework of iterative graph cuts based active contour. In IINBBGC method, an iterative narrowband \( R_0 \) around the evolving curve is obtained by morphological expansion. The pixels inside \( R_0 \) are regarded as the nodes of graph \( G \), and the nodes with label \( x_p = 1 \) and \( x_p = 0 \) correspond to the source \( S \) and sink \( T \), respectively. The weights of n-links connecting adjacent nodes are calculated using Eq. (5), and the weights of t-links of nodes on the inner and outer boundaries are set to infinity, and the weights of other i-links are computed using Eqs. (6) and (7) respectively.

\[ w_{pq} = \frac{\alpha_{pq}}{1 + \beta |I(p) - I(q)|} \]

\[ w_{ip} = |C_i(p) - I(p)| \]

\[ w_{pt} = |C_T(p) - I(p)| \]

where \( I \) is the image, \( I(p) \) and \( I(q) \) are intensity values of \( p \) and \( q \) respectively, and \( \beta > 0 \) is a weighted coefficient of the difference between \( I(p) \) and \( I(q) \). While \( \alpha_{pq} \) is the weight of vector in the neighborhood system, \( C_i(p) \) and \( C_T(p) \) are the means of inner and outer
regions of the closed curve $C$, respectively. If the probability density functions $h_{y}(l(p))$ and $h_{x}(l(p))$ of the inner and outer regions are used, the weights of t-links also can be assigned as

\begin{align}
\text{w}_{xp} &= -\log (h_{y}(l(p))) \\
\text{w}_{pt} &= -\log (h_{x}(l(p)))
\end{align}

(8) (9)

3. The proposed model

3.1. The discrete formulation

From Refs. [24], the general formulation of additive GC based ACMs in a narrow band framework can be written as follows:

\begin{equation}
\begin{cases}
E = E_{b}(p, q) + E_{r}(p) \\
p, q \in R_{NB}
\end{cases}
\end{equation}

(10)

where $R_{NB}$ is the iterative narrow band, $p, q$ indicate pixels in $R_{NB}$, $E_{b}(p, q)$ and $E_{r}(p)$ are the discrete GC formulations of boundary and region forces respectively.

Similar to the discrete representation of GAC in 1INBBGC method, the term $E_{b}(p, q)$ is used to measure the cost of assigning different labels to adjacent pixels $p$ and $q$, which can be defined as follows:

\begin{equation}
E_{b}(p, q) = \sum_{p, q \in R_{NB}} \sum_{p \in N(q)} \alpha_{pq} \left( \delta_{xp} x_{p} + \delta_{xp} x_{q} \right) e^{-\|l(p) - l(q)\|}
\end{equation}

(11)

where $x_{p}$ is the binary label of pixel $p$, $l$ is the image, $l(p)$ and $l(q)$ are the intensity values of pixels $p$ and $q$ respectively, $N$ is the neighborhood system which is generally 4-neighborhood or 8-neighborhood, $\alpha_{pq} = \frac{\delta^{2}}{2\|e_{pq}\|}$, $\delta$ is the mesh size, $\|e_{pq}\|$ is the length of edge vector $e_{pq}$. $\delta_{pq}$ is the angle between adjacent vectors. The illustration of 4-neighborhood and 8-neighborhood systems is shown in Fig. 1. If $l$ is set to 1 unit, for 4-neighborhood, $\Delta_{pq} = \frac{\pi}{2}$, $|e_{1}| = |e_{2}| = 1$, $\omega_{1} = \omega_{2} = \frac{\pi}{2}$, and for 8-neighborhood, $\Delta_{pq} = \frac{\pi}{4}$, then $|e_{1}| = |e_{3}| = 1$, $|e_{2}| = |e_{4}| = \sqrt{2}$, $\omega_{1} = \omega_{3} = \frac{\pi}{4}$, $\omega_{2} = \omega_{4} = \frac{\pi}{2}$.

The coefficient $K_{pq}$ in Eq. (11) is an adaptive adjustment used to adjust the boundary term in different cases. It is assumed that the more similar pixel $p$ and $q$ are, the larger the weight of edge connecting them should be, otherwise the weight is smaller. So $K_{pq}$ is designed as follows:

\begin{equation}
K_{pq} = e^{1 + \text{sign}(p)\text{sign}(q) - \mu_{pq}}
\end{equation}

(12)

where $\text{sign}(\cdot)$ is the symbolic function, $s(\cdot)$ and $\mu_{pq}$ are the signed pressure function and constraint variable respectively defined as follows

\begin{equation}
s(p) = \frac{f_{i}(p) + f_{j}(p)}{2}
\end{equation}

(13)

\begin{equation}
\mu_{pq} = \begin{cases}
\min(l_{int}, l_{ext}), \text{sign}(s(p)) \neq \text{sign}(s(q)) \\
\max(l_{int}, l_{ext}), \text{sign}(s(p)) = \text{sign}(s(q))
\end{cases}
\end{equation}

(14)

In Eq. (13), $f_{i}(p)$ and $f_{j}(p)$ are local means of the interior and exterior neighborhoods of pixel $p$ respectively, calculated using Eqs. (15) and (16) in which $K_{p}$ is a Gaussian kernel filter with the variance of $\sigma$, $\phi(p)$ is a binary signed distance function to the evolving curve, $H_{e}$ is a Heaviside function with parameter $e$, and $\ast$ is a convolution operation.

\begin{equation}
f_{i}(p) = \frac{K_{p}(p)^{\ast}[H_{e}(\phi(p))l(p)]}{K_{p}(p)^{\ast}H_{e}(\phi(p))}
\end{equation}

(15)

\begin{equation}
f_{j}(p) = \frac{K_{p}(p)^{\ast}[1 - H_{e}(\phi(p))l(p)]}{K_{p}(p)^{\ast}[1 - H_{e}(\phi(p))]} \end{equation}

(16)

In Eq. (14), $L_{int}$ and $L_{ext}$ represent two cost functions of assigning different labels for pixels $(p, q)$ respectively, such as $(x_{p} = 0, x_{q} = 1)$ and $(x_{p} = 1, x_{q} = 0)$. $L_{int}$ and $L_{ext}$ indicate two cost functions of assigning same labels for pixels $(p, q)$, such as $(x_{p} = 0, x_{q} = 0)$ and $(x_{p} = 1, x_{q} = 1)$ for $(p, q)$ respectively, which are defined as follows:

\begin{equation}
L_{int} = \left| (l(p) - f_{i}(p))^{2} - (l(q) - f_{i}(q))^{2} \right|
\end{equation}

(17)

\begin{equation}
L_{ext} = \left| (l(p) - f_{i}(p))^{2} - (l(q) - f_{i}(q))^{2} \right|
\end{equation}

(18)

\begin{equation}
L_{int} = \left| (l(p) - f_{j}(p))^{2} + (l(q) - f_{j}(q))^{2} \right|
\end{equation}

(19)

\begin{equation}
L_{ext} = \left| (l(p) - f_{j}(p))^{2} + (l(q) - f_{j}(q))^{2} \right|
\end{equation}

(20)

Based on the KL distance of the inner and outer neighborhoods, the discrete formulation of region term $E_{r}(p)$ used to measure the cost of assigning label to pixel $p$ in a narrow band graph is defined as follows:

\begin{equation}
E_{r}(p) = \sum_{p \in R_{NB}} d_{i}(p) \log \left( \frac{d_{i}(p)}{d_{o}(p)} \right) x_{p} + \sum_{p \in R_{NB}} d_{o}(p) \log \left( \frac{d_{o}(p)}{d_{i}(p)} \right) (1 - x_{p})
\end{equation}

(21)

where $d_{i}(p)$ and $d_{o}(p)$ are the probability distribution functions of the inner and outer neighborhoods of pixel $p$ respectively. Assuming that the intensity of local region obeys the Gaussian distribution, $d_{i}(p)$ and $d_{o}(p)$ can be expressed as follows:

\begin{equation}
d_{i}(p) = \frac{1}{\sqrt{2\pi\sigma_{i}(p)}} \exp \left( -\frac{(l(p) - f_{i}(p))^{2}}{2\sigma_{i}^{2}(p)} \right)
\end{equation}

(22)

\begin{equation}
d_{o}(p) = \frac{1}{\sqrt{2\pi\sigma_{o}(p)}} \exp \left( -\frac{(l(p) - f_{j}(p))^{2}}{2\sigma_{o}^{2}(p)} \right)
\end{equation}

(23)

where $\sigma_{i}(p)$ and $\sigma_{o}(p)$ corresponding to the standard deviations of the inner and outer neighborhoods respectively are calculated as follows:

\begin{equation}
\sigma_{i}^{2}(p) = \frac{\int k_{o}(y - p) (l(y) - f_{i}(p))^{2} H_{e}(\phi(y)) \, dy}{\int k_{o}(y - p) H_{e}(\phi(y)) \, dy}
\end{equation}

(24)

\begin{equation}
\sigma_{o}^{2}(p) = \frac{\int k_{o}(y - p) (l(y) - f_{j}(p))^{2} (1 - H_{e}(\phi(y))) \, dy}{\int k_{o}(y - p) (1 - H_{e}(\phi(y))) \, dy}
\end{equation}

(25)

3.2. The construction of the narrow band graph

For local segmentation, a narrow band graph should be constructed and the narrow band should contain the desired boundary. The construction of the network graph $G$ is shown in Fig. 2. The red curve in Fig. 2 is the evolving curve $C$, the inner green curve $C_{in}$ is obtained using morphological erosion, and the outer green $C_{out}$ is near the image boundary. The region between $C_{in}$ and $C_{out}$ is the narrow band $R_{NB}$. Each pixel in the graph $G$ is assigned a binary label. The pixels outside $C_{out}$ with label $x_{p} = 0$ are regarded as background ($B$) corresponding to the source node set S, and the pixels
inside $C_{in}$ with $x_p = 1$ are regarded as object ("0") corresponding to the sink set $T$. The corresponding relationship is shown in Table 1.

Using the 8-neighborhood system of the nodes in the graph $G$, the weights of n-links connecting adjacent nodes are set as the boundary term $E_{ij}(p,q)$. The weights of t-links connecting the nodes on curve $C_{in}$ and $C_{out}$ with terminal nodes are set to infinity, and derived from region term $E_{r}(p)$, the weights of others t-links connecting the pixels in the narrow band with terminal set $S$ and $T$ should be assigned as follows:

$$w_{pt} = d_t(p) \log \left( \frac{d_t(p)}{d_t(q)} \right)$$ (26)

$$w_{sp} = d_s(p) \log \left( \frac{d_s(p)}{d_s(q)} \right)$$ (27)

3.3. Implementation steps of the proposed local segmentation model

The main steps of the proposed KLD-GCBAC model for local segmentation can be summarized as follows:

1. Place initial closed curve $C$ inside the object to be segmented. The binary label $x_p = 1$ if the pixel is inside $C$, otherwise $x_p = 0$. The boundary of the image or any closed curve containing the object boundary is selected as the fixed outer boundary $C_{out}$ of the narrow band.
2. Carry out the morphological erosion of curve $C$ to obtain the inner boundary $C_{in}$ of the narrowband. The region between $C_{in}$ and $C_{out}$ is taken as the narrow band, as shown in Fig. 2.
3. Calculate $f_I(p), f_O(p)$ using Eqs. (15),(16) and $\sigma_I^2(p), \sigma_O^2(p)$ using Eqs. (24),(25), respectively.
4. Map the image to a network graph according to previously methods described in Section 3.2, and assign weights for n-links and t-links.

5. Use the max-flow/min-cut algorithm [15] to realize the minimum cut of the network graph, and update the labels of pixels in the narrowband to obtain current segmentation result.
6. Smooth the curve using morphological closing operation.
7. Repeat steps (2)-(6) until convergence.

4. Experimental results

We apply the proposed KLD-GCBAC model on several synthetic and medical images to validate its local segmentation effectiveness, comparing with GCBAC [21], GCBCV [19], INNBGC [22] and MLM-GCACM [26] models. For GCBAC model, the size of radius of disk-type structure element for morphological expansion operations is 2. For GCBCV model, the size of rectangular window is 31*31, $\mu = 0.2 \times 255 \times 255$. For INNBGC model, the weights of t-links are set using Eqs. (6),(7) and $\mu$ is generally set to 0.1. And for MLM-GCACM model, the standard deviation $\sigma$ of Gaussian filter is 10. Other parameters need to be set respectively for different images. All models use 8-neighborhood system, and these experiments are implemented using Matlab R2015b on a PC of 3.4 GHz Intel (R) Core (TM) CPU. In all experimental results, the green curves are initial contours and the red curves are final segmentation results.

Using the same initial contour, the comparison of local segmentation results using those five models on two single-object images, two multi-object synthetic images and four medical images are shown in Figs. 3–5, respectively. It can be seen that, for the weights of edges are inversely proportional to the gradient of the image, GCBAC can easily capture the edges with large gradient, but falls into the weak boundary. GCBCV model with global segmentation characteristics cannot realize local segmentation from multi-object images. INNBGC and MLM-GCACM based on iterative narrowband and graph cuts can obtain local segmentation results, however, the weights of t-links in INNBGC model cannot describe the local features of the image well, which makes some segmentation results unsatisfactory; and the narrow band in MLM-GCACM model is constructed based on the entire image, so GCBCV model with global segmentation characteristics cannot realize local segmentation from multi-object images. INNBGC and MLM-GCACM based on iterative narrowband and graph cuts can obtain local segmentation results, especially for medical images with complex background and weak edges in Fig. 5.

There is a common indicator to measure the segmentation accuracy, i.e., the Jaccard similarity (JS), which is defined as follows:

$$JS = \frac{|V_{gold} \cap V_{algorithm}|}{|V_{gold} \cup V_{algorithm}|}$$ (28)

where $V_{gold}$ and $V_{algorithm}$ indicate the manually segmented “gold standard” by experts and actual segmentation result respectively, and $|I|$ is the operator for calculating the number of pixels within the conditional region. Then, the bigger JS is, the more accurate the segmentation result is. The comparison of JS coefficients between “gold standards” and segmentation results in Fig. 5 is shown in Table 2, and the comparison of required iterations number and running time is shown in Table 3, in which case 1 to case 4 correspond to from the first to fourth image in Fig. 5, respectively. It can be seen that except for case 4, the proposed KLD-GCBAC has better segmentation results with highest accuracy. For case 1, case 2 and case 4, the iterations number and running time required of GCBAC is the least, which is mainly due to the stopping of curve evolution in advance resulting in rapid convergence. For the iterative con-
Table 2  
Comparison of JS coefficients between “gold standards” and segmentation results for experiments in Fig. 5.

<table>
<thead>
<tr>
<th></th>
<th>GCBAC</th>
<th>GCBCV</th>
<th>IINBBGC</th>
<th>MLM-GCACM</th>
<th>KLD-GCBAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>0.0578</td>
<td>0.3658</td>
<td>0.1374</td>
<td>0.8658</td>
<td>0.9065</td>
</tr>
<tr>
<td>case 2</td>
<td>0.2588</td>
<td>0.6408</td>
<td>0.0514</td>
<td>0.8489</td>
<td>0.8645</td>
</tr>
<tr>
<td>case 3</td>
<td>0.1444</td>
<td>0.9373</td>
<td>0.9252</td>
<td>0.9469</td>
<td>0.9505</td>
</tr>
<tr>
<td>case 4</td>
<td>0.0202</td>
<td>0.0084</td>
<td>0.7532</td>
<td>0.9861</td>
<td>0.9763</td>
</tr>
</tbody>
</table>

Table 3  
Comparison of required iterations number/running time (s) for experiments in Fig. 5.

<table>
<thead>
<tr>
<th></th>
<th>GCBAC</th>
<th>GCBCV</th>
<th>IINBBGC</th>
<th>MLM-GCACM</th>
<th>KLD-GCBAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>1 / 0.41</td>
<td>14 / 1.26</td>
<td>40 / 4.11</td>
<td>8 / 1.37</td>
<td>6 / 1.02</td>
</tr>
<tr>
<td>case 2</td>
<td>1 / 0.28</td>
<td>4 / 0.64</td>
<td>38 / 4.19</td>
<td>8 / 1.42</td>
<td>5 / 0.94</td>
</tr>
<tr>
<td>case 3</td>
<td>12 / 1.33</td>
<td>9 / 1.02</td>
<td>136 / 45.71</td>
<td>17 / 2.94</td>
<td>2 / 0.87</td>
</tr>
<tr>
<td>case 4</td>
<td>1 / 0.33</td>
<td>14 / 1.25</td>
<td>112 / 23.05</td>
<td>17 / 2.75</td>
<td>2 / 0.57</td>
</tr>
</tbody>
</table>

The proposed MLM-GCACM model can achieve better segmentation results and has higher operating efficiency.

5. Discussion

In the proposed KLD-GCBAC model, the standard deviation $\sigma$ of Gaussian kernel function is an important parameter. Since the convolution has a certain smoothing effect, its value determines the size of local neighborhood, and has an effect on the extraction of local features and the weights of n-links.

Keeping other parameters remain unchanged, the segmentation results of KLD-GCBAC on a synthetic image and a MR brain image with different $\sigma$ are shown in Figs. 6 and 7 respectively. It can be seen that if $\sigma$ is too small, the evolving curve easily falls into the minimum within the object; and when $\sigma$ is too large, the smoothing effect can easily make the curve cross the real boundary. In Fig. 6, the suitable value of $\sigma$ in which the correct segmentation result can be obtained is $0.5 < \sigma < 2$, while in Fig. 7, the result shown as in Fig. 7(d) can be obtained if $\sigma = 0.78 < \sigma < 1.4$. Thus, medical images with complex background and weak edges are more sensitive to $\sigma$ than synthetic images.

KLD-GCBAC model uses morphological closing operation to eliminate small voids within the evolution curve during the iteration process and smooth the curve boundary. The radius $r$ of the disk structure element also has an effect on the local segmentation result. Fig. 8 shows the comparison of local segmentation results of a MR left ventricular image and a real image with different $r$ in...
which $r = 0$ corresponds to the situation without smoothing operation. From the first row of Fig. 8, it can be seen that on the absence of morphological closing operation, the evolving curve may converge to local weak boundary. Adding morphological operations is helpful for the evolution of the curve. But in the second row of Fig. 8, there are obvious boundaries outside the object, and the best segmentation result is obtained without smoothness operation. With the value of $r$ increases, error segmentation across the real boundary becomes more and more serious.

The narrow band of the proposed model is constructed between the erosion curve and the outer boundary of the image, and the evolving curve is placed inside the object to expand outward until
stop at the real boundary, then the desired result should be included in the narrow band. Fig. 9 shows the comparison of segmentation results of a real image using different initial contours. It can be seen that correct result can be obtained in Fig. 9(b), (c) and (d), in which the initial contour is inside the object completely or very close to the boundary, a rectangle or a pixel. In these situations, the real boundary can be achieved in the narrow band. But part of the inter curve of the narrow band using morphological erosion is already outside the object in Fig. 9(a), which leads to error segmentation.

6. Conclusions

The graph cuts and local statistical information based KLD-GCBAC model proposed in this paper is a local segmentation model. It uses the iterative narrowband to construct a network graph, and designs the weights of n-links and t-links combining the local means and KL distance of the inner and outer neighborhood regions, which can better describe the local features of the image. The experimental results on inhomogeneous synthetic and medical images verify the local segmentation ability of KLD-GCBAC model with high accuracy and operating efficiency.

Acknowledgements

This work was supported by the Fundamental Research Funds for the Central Universities (China), National Natural Science Foundation of China (Nos. 81671848 and 81371635) and Key Research and Development Project of Shandong Province (No. 2016GGX101017).

References


